CUBE ROOT WEAK CONVERGENCE OF
EMPIRICAL ESTIMATORS OF A DENSITY LEVEL SET

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Given n independent random vectors with common density f on $\mathbb{R}^d$, we study the weak convergence of three empirical-measure based estimators of the convex λ-level set L_λ of f. These three estimators are the excess mass set, the minimum volume set and the maximum probability or modal set, all selected from a class of convex sets $\mathcal{A}$ that contains L_λ. Since these set-valued estimators approach L_λ, even the formulation of weak convergence is non-standard. Under regularity assumptions on f and $\mathcal{A}$ we establish weak convergence of the symmetric difference of each of the three estimators and L_λ, at rate $1/n^{1/3}$. It turns out that the minimum volume set and the maximum probability set are asymptotically indistinguishable, whereas the excess mass set exhibits "richer" limit behavior. Arguments rely on the boundary local empirical process, its cylinder representation, dimension-free concentration around the boundary of L_λ, and the set-valued argmax of a drifted Wiener process.

This is joint work in progress with Philippe Berthet (Toulouse).